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Squeezing Effects of a Mesoscopic Dissipative Coupled Circuit

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We study the quantum effect of a mesoscopic dissipative-coupled *RLC* circuit of the capacitances. We find that if the quantum dissipative system is in the vacuum state at the initial time, it will evolve to a squeezed coherent state under the effect of an external pulse source because of the presence of the coupling and damping.

KEY WORDS: dissipative mesoscopic circuit; quantum fluctuation; squeezed states; coupling and damping; squeezing angle.

1. INTRODUCTION

Owing to the development of nanometer techniques and microelectronics, the trend of the miniaturization of integrated circuits and electronic components towards atomic scale becomes stronger and stronger. When the phase coherence length of the charge-carrier approaches the Fermi wavelength, quantum effects must be considered (Li and Chen, 1996a,b). Many researches on quantum effects in nondissipative mesoscopic circuits have been done (Lei *et al.*, 2001; Yu *et al.*, 1998; Yu and Liu, 1998).

Recently, some scholars have studied the influence of a resistance whose macroscopic parameter is R on the quantum effect in the dissipative mesoscopic circuit (Wang *et al.*, 2000; Zhang *et al.*, 1998). In these previous works, it can be assumed that the classical equation of motion of the *RLC* circuit is exactly the same as that of a damped harmonic oscillator.

There is no doubt that the *RLC* circuit is a simplest example of the dissipative system. Feynman and Vernon have come up with a theory of the mutual action between the quantum system and environment. In their theory, they suggest that the oscillators are in a bath to describe the system interacting with the environment. According to this theory, Calderia and Leggett (Sun and Yu, 1995;

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Yu and Sun, 1994) consider a damping harmonic oscillator as a dissipative system in which the harmonic oscillator couples to an environment that can be looked upon as a bath of many harmonic oscillators. On this base, they obtained the effective Hamiltonian of a damped harmonic oscillator system that is now called the Calderia–Kani (CK) Hamiltonian.

As everyone knows, the resistance of a circuit results from the scattering of conduction electrons by the crystal lattice, and the lattice vibration is equivalent to a group of harmonic oscillators. So the mesoscopic *RLC* circuit can be regarded as an interactive system composed of an electromagnetic harmonic oscillator coupled to a bath of lattice oscillators (Ji *et al.*, 2002). Therefore the effective Hamiltonian of the *RLC* circuit is the CK Hamiltonian. With the canonical commutation relation $[q, p] = i\hbar$, this Hamiltonian automatically yields the classical motion equation of the *RLC* circuit.

We study the quantum effects of a mesoscopic dissipative *RLC* circuit coupled with a capacitance in this paper. We will suggest the Hamiltonian of the dissipative-coupled circuit. Using the canonical quantization method from the classical motion equations, we obtain the quantum fluctuations of charge and current in the squeezing state. Furthermore, we have investigated the squeezing effect of the circuit.

2. DIAGONALIZE HAMILTONIAN

Now, we study two meshes of the *RLC* coupled circuit of capacitors (see Fig. 1). It is well known that the classical motion equations of the mutual-inductance circuit with a source are

$$L_1\ddot{q}_1 + R_1\dot{q}_1 + C^{-1}(q_1 - q_2) + C_1^{-1}q_1 = \varepsilon(t), \tag{1}$$

$$L_2\ddot{q}_2 + R_2\dot{q}_2 - C^{-1}(q_1 - q_2) + C_2^{-1}q_2 = 0. \tag{2}$$

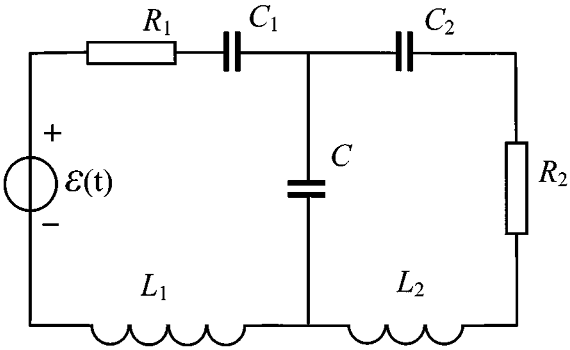

Fig. 1. Coupled circuit of capacitors.

where L_j, R_j, and C_j stand for inductance, resistance, and capacitance of two meshes, respectively, and C is the common capacitance parameter; and q_j represents the electric charge.

To simplify the problem, we suppose that

$$R_1 L_1^{-1} = R_2 L_2^{-1} = 2\beta. \tag{3}$$

According to the Calderia and Leggett's quantization scheme for a *RLC* circuit, when $\varepsilon(t) = 0$, we can obtain the classical Hamiltonian of this system from above equations.

$$H(q) = e^{-2\beta t}\left[\frac{p_1^2}{2L_1} + \frac{p_2^2}{2L_2}\right] + e^{2\beta t}\left[\frac{q_1^2}{2C_1} + \frac{q_2^2}{2C_2} + \frac{(q_1 - q_2)^2}{2C}\right]. \tag{4}$$

From the Hamiltonian, the conjugate momenta p_j can be gotten as follows

$$p_j = e^{2\beta t} L_j \dot{q}_j \quad (j = 1, 2),$$

where the variables q_j and p_j play the part of the generalized coordinates and momenta, respectively. Equation (4) represents a pair of damping harmonic oscillators which are coupling each other. To simplify the Hamiltonian equation (4), now, let us transform the variables (q_j, p_j) into the variables (Q_j, P_j). The transformations of the coordinate and momenta (Lei *et al.*, 2001; Zhang *et al.*, 2001) are

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = e^{\beta t}\begin{pmatrix} A\cos\varphi & -B\sin\varphi \\ A\sin\varphi & B\cos\varphi \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \tag{5}$$

$$\begin{aligned}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} &= e^{-\beta t}\begin{pmatrix} B\cos\varphi & -A\sin\varphi \\ B\sin\varphi & A\cos\varphi \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \\ &\quad + \beta\, e^{\beta t}\begin{pmatrix} \sqrt{L_1 L_2} & 0 \\ 0 & \sqrt{L_1 L_2} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix},\end{aligned} \tag{6}$$

where

$$B^4 = A^{-4} = L_1^{-1} L_2, \tag{7}$$

$$\cot(2\varphi) = \frac{C + C_1}{2C_1}\sqrt{\frac{L_2}{L_1}} - \frac{C + C_2}{2C_2}\sqrt{\frac{L_1}{L_2}}. \tag{8}$$

Then we can transform Eq. (4) into the separable form by using Eqs. (5)–(8), that is,

$$H(Q) = \frac{1}{2\sqrt{L_1 L_2}} P_1^2 + \frac{1}{2\sqrt{L_1 L_2}} P_2^2 + \frac{1}{2} k_1 Q_1^2 + \frac{1}{2} k_2 Q_2^2, \tag{9}$$

where

$$k_1 = B^2\left(C^{-1} + C_1^{-1}\right)\cos^2\varphi + A^2\left(C^{-1} + C_2^{-1}\right)\sin^2\varphi + C^{-1}\sin 2\varphi - \alpha_1\beta^2 \tag{10}$$

$$k_2 = B^2\left(C^{-1} + C_1^{-1}\right)\sin^2\varphi + A^2\left(C^{-1} + C_2^{-1}\right)\cos^2\varphi - C^{-1}\sin 2\varphi - \alpha_1\beta^2. \tag{11}$$

On the basis of the standard quantization principle we can quantize the system. First of all, we assume the following commutation relations are valid:

$$[\hat{Q}_j, \hat{P}_k] = i\hbar\delta_{jk}, \qquad [\hat{Q}_j, \hat{Q}_k] = [\hat{P}_j, \hat{P}_k] = 0 \quad (j, k = 1, 2).$$

Thus we can quantize the system by means of Q_j and P_k $(j, k = 1, 2)$. After quantization the Hamiltonian of system transforms into the form of operator, that is,

$$\hat{H}(Q) = \frac{1}{2\sqrt{L_1 L_2}}\hat{P}_1^2 + \frac{1}{2\sqrt{L_1 L_2}}\hat{P}_2^2 + \frac{1}{2}k_1\hat{Q}_1^2 + \frac{1}{2}k_2\hat{Q}_2^2. \tag{12}$$

Obviously, above formula represents two independent quantum harmonic oscillators. Therefore we may easily obtain the quantum energy levels of the system, coupled *RLC* circuit,

$$E = \hbar\Omega_1\left(n_1 + \frac{1}{2}\right) + \hbar\Omega_2\left(n_2 + \frac{1}{2}\right), \quad (n_1, n_2 = 0, 1, 2, \ldots), \tag{13}$$

where

$$\Omega_1^2 = \frac{k_1}{\sqrt{L_1 L_2}}, \qquad \Omega_2^2 = \frac{k_2}{\sqrt{L_1 L_2}}. \tag{14}$$

For an independent mesoscopic *RLC* circuit without any mutual coupling, if it is in the vacuum state at the initial time, it will evolve to a coherent state under the effect of an external source. For a mesoscopic coupled circuit, if the system is in the vacuum state at the initial time, it will evolve to a squeezed coherent state because of presence of the coupling, which had been described in Lei *et al.* (2001). Comparing Eqs. (5) and (6), we can see that the transformation contains not only rotation transformation but also squeezing transformation. For example, as the factor $(L_1/L_2)^{1/4}$ appears in Q_1, it's the inverse $(L_2/L_1)^{1/4}$ appears in P_1. It means the squeezing of the charge and one of its canonical conjugate current are reversed each other. This squeezing originates from the coupling effect and damping. So, when a pulse signal with a finite amplitude enters the mesoscopic circuit at $t = 0$, if the pulse width $\tau \to 0$, the circuit will evolve to a squeezed coherent state from its initial vacuum state.

3. QUANTUM FLUCTUATION

The squeezed vacuum state $|0,0\rangle_{r_1,r_2}$ takes the following form in the particle number representation

$$|0,0\rangle_{r_1,r_2} = \sec h^{1/2}(r_1)\sum_{n=0}^{\infty}\frac{-e^{i\theta_1}\ \tanh(r_1)^n[(2n)!]^{1/2}}{n!2^n}|2n\rangle_1$$
$$\otimes\ \sec h^{1/2}(r_2)\sum_{m=0}^{\infty}\frac{-e^{i\theta_2}\ \tanh(r_2)^m[(2m)!]^{1/2}}{m!2^m}|2m\rangle_2, \qquad (15)$$

where r_j and $\theta_j (j = 1, 2)$ stand for the squeezing magnitude and phase parameters. In this squeezed vacuum state, the mean value and mean square value of charges Q_j and their conjugate variables P_j are, respectively,

$$\overline{Q_j} = 0, \qquad \overline{P_j} = 0, \qquad (16)$$

$$\overline{Q_j^2} = \frac{\hbar}{2\Omega_j\sqrt{L_1L_2}}\ \sec h(r_j)$$
$$\times\sum_{n=0}^{\infty}\frac{(2n)!\tanh^{2n}(r_j)\cdot[4n+1-2(2n+1)\cos(\theta_j)\tanh(r_j)]}{(n!)^2 2^{2n}} \qquad (17)$$

$$\overline{P_j^2} = \frac{\Omega_j\hbar\sqrt{L_1L_2}}{2}\ \sec h(r_j)$$
$$\times\sum_{n=0}^{\infty}\frac{(2n)!\tanh^{2n}(r_j)\cdot[4n+1+2(2n+1)\cos(\theta_j)\ \tanh(r_j)]}{(n!)^2 2^{2n}}$$
$$(j = 1, 2). \qquad (18)$$

Using Eqs. (5), (6), and (16)–(18), we may obtain the mean value and the mean square value of charges q_j and their conjugate variables p_j, respectively, in the squeezed state

$$\overline{q_1} = \overline{q_2} = 0, \qquad \overline{p_1} = \overline{p_2} = 0, \qquad (19)$$

$$\overline{(\Delta\hat{q}_1)^2} = e^{-2\beta t}\frac{\hbar}{2L_1}\left(x_1\frac{\cos^2\varphi}{\Omega_1} + x_2\frac{\sin^2\varphi}{\Omega_2}\right), \qquad (20)$$

$$\overline{(\Delta\hat{q}_2)^2} = e^{-2\beta t}\frac{\hbar}{2L_2}\left(x_1\frac{\sin^2\varphi}{\Omega_1} + x_2\frac{\cos^2\varphi}{\Omega_2}\right), \qquad (21)$$

$$\overline{(\Delta\hat{p}_1)^2} = e^{2\beta t}\frac{\hbar L_1}{2}\left[y_1\left(\Omega_1+\beta^2\Omega_1^{-1}\right)\cos^2\varphi + y_2\left(\Omega_2+\beta^2\Omega_2^{-1}\right)\sin^2\varphi\right], \qquad (22)$$

$$\overline{(\Delta\hat{p}_2)^2} = e^{2\beta t}\frac{\hbar L_2}{2}\left[y_1\left(\Omega_1+\beta^2\Omega_1^{-1}\right)\sin^2\varphi + y_2\left(\Omega_2+\beta^2\Omega_2^{-1}\right)\cos^2\varphi\right], \qquad (23)$$

where

$$x_j = \sec h(r_j) \sum_{n=0}^{\infty} \frac{(2n)! \tanh^{2n}(r_j) \cdot [4n + 1 - 2(2n+1)\cos(\theta_j)\tanh(r_j)]}{(n!)^2 2^{2n}}, \tag{24}$$

$$y_j = \sec h(r_j) \sum_{n=0}^{\infty} \frac{(2n)! \tanh^{2n}(r_j) \cdot [4n + 1 + 2(2n+1)\cos(\theta_j)\tanh(r_j)]}{(n!)^2 2^{2n}}, \quad (j = 1, 2). \tag{25}$$

It can be seen that x_j and y_j represent the dimensionless parameters of charges and their conjugate variables, respectively. When there is no power in the circuit, it can be found from above equations that the averages of the charges q_j and their conjugate variables p_j are zero, but their mean square values are not all zero in the squeezed vacuum state. And the uncertainty relation of mesh is

$$\overline{(\Delta q_1)^2(\Delta p_1)^2} \neq \overline{(\Delta q_2)^2(\Delta p_2)^2}$$

4. SQUEEZED EFFECT

Lei *et al.* (2001) and Fan and Pan (1998) have pointed out that the squeezing effect in nondissipative circuit originates from the coupling effect. The squeezing magnitude parameters are connected with the circuit parameters and the coupling parameter as well. So, we may obtain different squeezing by controlling both the circuit parameters and the coupling parameter. The changes of x_j and y_j with the squeezed magnitude r are drawn in Figs. 2. and 3.

These curves clearly indicate that when $\theta_j = 0$, x_j decreases with increase of r_j, but when $\theta_j = \pi$, x_j increases. And yet the changes of y_j are just contrary to x_j. As for the dissipative coupling circuits, from Eqs. (20) to (23), we may observe that the squeezing effect results from not only the coupling but also the damping.

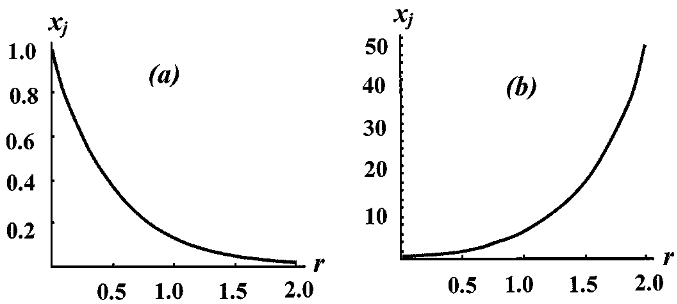

Fig. 2. Changes of x_j vs. the squeezed magnitude r (a) $\theta = 0$, (b) $\theta = \pi$.

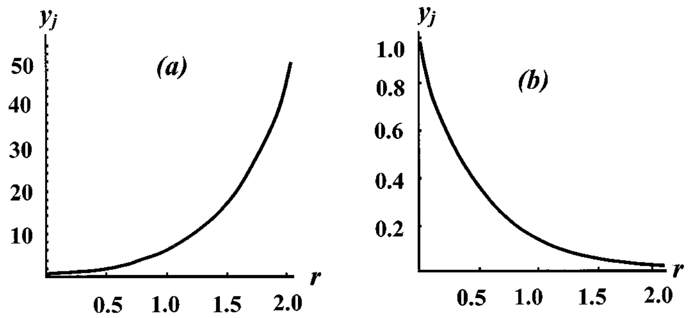

Fig. 3. Changes of y_j vs. the squeezed magnitude r (a) $\theta = 0$, (b) $\theta = \pi$.

And the latter is more important because the influence of damping factor upon the squeezing effect is an exponent's function. If the circuit parameters are all fixed, the damping factor becomes the only one influencing upon the squeezing effect of the circuit. Now we consider a particular situation, when $r_j = 0$ and the dissipative system is in a vacuum state, the quantum fluctuations of the charge and current are respectively

$$\overline{(\Delta \hat{q}_1)^2} = e^{-2\beta t} \frac{\hbar}{2L_1} \left(\frac{\cos^2 \varphi}{\Omega_1} + \frac{\sin^2 \varphi}{\Omega_2} \right), \tag{26}$$

$$\overline{(\Delta \hat{q}_2)^2} = e^{-2\beta t} \frac{\hbar}{2L_2} \left(\frac{\sin^2 \varphi}{\Omega_1} + \frac{\cos^2 \varphi}{\Omega_2} \right), \tag{27}$$

$$\overline{(\Delta \hat{p}_1)^2} = e^{2\beta t} \frac{\hbar L_1}{2} \left[\left(\Omega_1 + \beta^2 \Omega_1^{-1} \right) \cos^2 \varphi + \left(\Omega_2 + \beta^2 \Omega_2^{-1} \right) \sin^2 \varphi \right], \tag{28}$$

$$\overline{(\Delta \hat{p}_2)^2} = e^{2\beta t} \frac{\hbar L_2}{2} \left[\left(\Omega_1 + \beta^2 \Omega_1^{-1} \right) \sin^2 \varphi + \left(\Omega_2 + \beta^2 \Omega_2^{-1} \right) \cos^2 \varphi \right]. \tag{29}$$

If the circuit parameters and coupling parameter are fixed, the all factors, except for the exponent factor, are constant in the above equations. With the increase of time, the quantum fluctuation of charge decreases but the quantum fluctuation of current increases continuously. Their uncertainty relations are

$$\overline{(\Delta q_1)^2 (\Delta p_1)^2} = \overline{(\Delta q_2)^2 (\Delta p_2)^2}$$

which has nothing to do with time and kept unchanged.

At the squeezed vacuum state, generally speaking, when the circuit parameters and coupling parameter are fixed, the squeezing angle is regulated to control the squeezing effect. From Eqs. (20) to (25) as well as Figs. 2 and 3, we may find out that when $\theta_j = 0$, the coupling and damping affect the quantum fluctuation in the same direction, a much deeper squeezing is gained. But when $\theta_j = \pi$, the coupling will destroy the squeezing of charge originating from the damp. As for the quantum fluctuation of current, we may gain a similar influence, but the result is just contrary.

5. CONCLUSION

Our study indicates that when the circuit parameters and coupling parameter are totally fixed, the quantum dissipative system will evolve to a squeezed coherent state from the initial vacuum state under the effect of an external source. The squeezing effect originates from the coupling effect and damping, and it is related to the squeezing angle.

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